

# PERTURBATIVE ANALYSIS OF CHERN-SIMONS FIELD THEORY IN THE COULOMB GAUGE

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## Abstract

In this paper we analyse the perturbative aspects of Chern-Simons field theories in the Coulomb gauge. We show that in the perturbative expansion of the Green functions there are neither ultraviolet not infrared divergences. Moreover, all the radiative corrections are zero at any loop order. Some problems connected with the Coulomb gauge fixing, like the appearance of spurious singularities in the computation of the Feynman diagrams, are discussed and solved. The regularization used here for the spurious singularities can be easily applied also to the Yang-Mills case, which is affected by similar divergences.

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# 1 Introduction

In the recent past, the Chern–Simons (C–S) field theories [1, 2] have been intensively studied in connection with several physical and mathematical applications [3, 4]. A convenient gauge fixing for these theories is provided by the Coulomb gauge. As a matter of fact, despite of the presence of nontrivial interactions in the gauge fixed action, the calculations become considerably simpler than in the covariant gauges and a perturbative approach is possible also on non-flat manifolds [5]. Moreover, the dependence on the time in the Green functions is trivial, so that the C–S field theories can be treated in practice as two dimensional models. Starting from the seminal works of refs. [2, 6] and [7], the Coulomb gauge has been already applied in a certain number of physical problems involving C–S based models [4], [8]–[11], but still remains less popular than the covariant and axial gauges. One of the main reasons is probably the fact that there are many perplexities concerning the use of this gauge fixing, in particular in the case of the four dimensional Yang–Mills theories [12]–[15]. Recently, also the consistency of the C–S field theories in the Coulomb gauge has been investigated using various techniques [5, 9, 16, 17], but so far a detailed perturbative analysis in the non-abelian case is missing. To fill this gap, the radiative corrections of the Green functions are computed here at any loop order and it is shown that they vanish identically. No regularization is needed for the ultraviolet and infrared divergences since, remarkably, they do not appear in the amplitudes. The present result agrees with the previous analysis of [17], in which the commutation relations between the fields are proved to be trivial using the Dirac’s canonical approach to constrained systems. It is important to notice that the absence of any quantum correction despite of the presence of nontrivial self-interactions in the Lagrangian is a peculiarity of the Coulomb gauge that cannot be totally expected from the fact that the theories under consideration are topological, as finite renormalizations of the fields and of the coupling constants are always possible. For instance, in the analogous case of the covariant gauges, only the perturbative finiteness of the C–S amplitudes has been shown [18] in a regularization independent way exploiting BRST techniques [19]. Indeed, a finite shift of the C–S coupling constant has been observed in the Feynman gauges by various authors [20, 21].

The material presented in this paper is divided as follows. In Section 2 the C–S field theories with  $SU(n)$  gauge group are quantized using the

BRST approach. The Coulomb gauge constraint is weakly imposed and the proper Coulomb gauge is recovered suitably choosing the gauge fixing parameter. The singularities that may appear in the perturbative calculations are studied in details. Ultraviolet divergences are predicted by the naive power counting, but it will be shown in Section 3 that they are absent in the perturbative expansions of the Green functions. Still there are spurious singularities, which arise because the propagators are undamped in the time direction. They are completely removed with the introduction of a cut off in the zeroth components of the momenta. In Section 3, the quantum contributions to the  $n$ -point correlation functions are derived at all orders in perturbation theory. The one loop case is the most difficult, as nontrivial cancellations occur among different Feynman diagrams. To simplify the calculations, a crucial observation is proved, which drastically reduces their number. The total contribution of the remaining diagrams is shown to vanish after some algebra. The gluonic 2-point function requires some care and it is treated separately. At two loop, instead, any single Feynman diagram is identically zero. The reason is that, in order to build such diagrams, some components of the propagators and of the vertices are required, which are missing in the Coulomb gauge. At higher orders, the vanishing of the Feynman diagrams is proved by induction in the loop number  $N$ . Finally, in the Conclusions some open problems and future developments are discussed.

## 2 Chern-Simons Field Theory in the Coulomb Gauge: Feynman Rules and Regularization

The C-S action in the Coulomb gauge looks as follows:

$$S_{CS} = S_0 + S_{GF} + S_{FP} \quad (1)$$

where

$$S_0 = \frac{s}{4\pi} \int d^3x \epsilon^{\mu\nu\rho} \left( \frac{1}{2} A_\mu^a \partial_\nu A_\rho^a - \frac{1}{6} f^{abc} A_\mu^a A_\nu^b A_\rho^c \right) \quad (2)$$

$$S_{GF} = \frac{is}{8\pi\lambda} \int d^3x \left( \partial_i A^{ai} \right)^2 \quad (3)$$

and

$$S_{FP} = i \int d^3x \bar{c}^a \partial_i \left( D^i [A] c \right)^a \quad (4)$$

In the above equations  $s$  is a dimensionless coupling constant and the vector fields  $A_\mu^a$  represent the gauge potentials. Greek letters  $\mu, \nu, \rho, \dots$  denote space-time indices, while the first latin letters  $a, b, c, \dots = 1, \dots, N^2 - 1$  are used for the color indices of the  $SU(n)$  gauge group with structure constants  $f^{abc}$ . The theory is considered on the flat space-time  $\mathbf{R}^3$  equipped with the standard euclidean metric  $g_{\mu\nu} = \text{diag}(1, 1, 1)$ . The total antisymmetric tensor  $\epsilon^{\mu\nu\rho}$  is defined by the convention  $\epsilon^{012} = 1$ . Finally,

$$D_\mu^{ab} [A] = \partial_\mu \delta^{ab} - f^{abc} A_\mu^c$$

is the covariant derivative and  $\lambda$  is an arbitrary gauge fixing parameter.

In eq. (1) the Coulomb gauge constraint is weakly imposed and the proper Coulomb gauge fixing<sup>1</sup>, given by:

$$\partial_i A^{ai} = 0 \quad i = 1, 2 \quad (5)$$

is recovered setting  $\lambda = 0$  in eq. (3).

The partition function of the CS field theory described by eq. (1) is:

$$Z = \int D A D \bar{c} D c e^{i S_{CS}} \quad (6)$$

and it is invariant under the BRST transformations listed below:

$$\begin{aligned} \delta A_\mu^a &= (D_\mu [A])^a \\ \delta \bar{c}^a &= \frac{s}{4\pi\lambda} \partial_i A^{ai} \\ \delta c^a &= \frac{1}{2} f^{abc} c^b c^c \end{aligned} \quad (7)$$

From (1), it is possible to derive the Feynman rules of C-S field theory in the Coulomb gauge. The components of the gauge field propagator  $G_{\mu\nu}^{ab}(p)$  in the Fourier space are given by:

$$G_{jl}^{ab}(p) = -\delta^{ab} \frac{4\pi\lambda}{s} \frac{p_i p_l}{\mathbf{p}^4} \quad (8)$$

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<sup>1</sup> From now on, middle latin letters like  $i, j, k, \dots = 1, 2$  will indicate space indices.

$$G_{j0}^{ab}(p) = \delta^{ab} \left( \frac{4\pi}{s} \epsilon_{0jk} \frac{p^k}{\mathbf{p}^2} - \frac{4\pi\lambda}{s} \frac{p_j p_0}{\mathbf{p}^4} \right) \quad (9)$$

$$G_{0j}^{ab}(p) = -\delta^{ab} \left( \frac{4\pi}{s} \epsilon_{0jk} \frac{p^k}{\mathbf{p}^2} + \frac{4\pi\lambda}{s} \frac{p_0 p_j}{\mathbf{p}^4} \right) \quad (10)$$

$$G_{00}^{ab}(p) = -\delta^{ab} \frac{4\pi\lambda}{s} \frac{p_0^2}{\mathbf{p}^4} \quad (11)$$

with  $\mathbf{p}^2 = p_1^2 + p_2^2$ , while the ghost propagator  $G_{gh}^{ab}(p)$  reads as follows:

$$G_{gh}^{ab}(p) = \frac{\delta^{ab}}{\mathbf{p}^2} \quad (12)$$

Finally, the three gluon vertex and the ghost-gluon vertex are respectively given by:

$$V_{\mu_1 \mu_2 \mu_3}^{a_1 a_2 a_3}(p, q, r) = -\frac{is}{3!4\pi} (2\pi)^3 f^{a_1 a_2 a_3} \epsilon^{\mu_1 \mu_2 \mu_3} \delta^{(3)}(p + q + r) \quad (13)$$

and

$$V_{\text{gh } i_1}^{a_1 a_2 a_3}(p, q, r) = -i(2\pi)^3 (q)_{i_1} f^{a_1 a_2 a_3} \delta^{(3)}(p + q + r) \quad (14)$$

In the above equation we have only given the spatial components of the ghost-gluon vertex. From eq. (4), it is in fact easy to realize that in the Coulomb gauge its temporal component is zero.

At this point, a regularization should be introduced in order to handle the singularities that may arise in the computations of the Feynman diagrams. The potential divergences are of three kinds.

1. Ultraviolet divergences (UV). The naive power counting gives the following degree of divergence  $\omega(G)$  for a given Feynman diagram  $G$ :

$$\omega(G) = 3 - \delta - E_B - \frac{E_G}{2} \quad (15)$$

with <sup>2</sup>

(a)  $\delta$  = number of momenta which are not integrated inside the loops

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<sup>2</sup>We use here the same notations of ref. [22]

- (b)  $E_B$  = number of external gluonic legs
- (c)  $E_G$  = number of external ghost legs

Eq. (15) shows that UV divergences are possible in the two and three point functions, both with gluonic or ghost legs. Moreover, there is also a possible logarithmic divergence in the case of the four point interaction among two gluons and two ghosts. In principle, we had to introduce a regularization for these divergences but in practical calculations this is not necessary. As a matter of fact, we will see in Section 3 that there are no UV divergences in the quantum corrections of the Green functions.

2. Infrared (IR) divergences. In the pure C-S field theories [2] there are no problems of infrared divergences. As a matter of fact, it can be seen from the Feynman rules written above that the IR behavior of the gluonic propagator is very mild ( $\sim \frac{1}{|\mathbf{p}|}$ ). The potentially more dangerous IR singularities due to the ghost propagator are screened by the presence of the external derivative in the ghost-gluon vertex (14). However, we notice that IR divergences appear in the interacting case. For instance, in three dimensional quantum electrodynamics coupled with a C-S term, the IR divergences have been discussed in refs. [1, 7].
3. Spurious divergences. These singularities appear because the propagators (8)–(12) are undamped in the time direction and are typical of the Coulomb gauge. To regularize spurious divergences of this kind, it is sufficient to introduce a cutoff  $\Lambda_0 > 0$  in the domain of integration over the variable  $p_0$ :

$$\int_{-\infty}^{\infty} dp_0 \rightarrow \int_{-\Lambda_0}^{\Lambda_0} dp_0 \quad (16)$$

The physical situation is recovered in the limit  $\Lambda_0 \rightarrow \infty$ .

As we will see, this regularization does not cause ambiguities in the evaluation of the radiative corrections at any loop order. In fact, the integrations over the temporal components of the momenta inside the loops turn out to be trivial and do not interfere with the integrations over the spatial components.

### 3 Perturbative Analysis

In this Section we compute the  $n$ -point correlation functions of C-S field theories at any loop order. To this purpose, we choose for simplicity the proper Coulomb gauge, setting  $\lambda = 0$  in eq. (3). In this gauge the gluon-gluon propagator has only two nonvanishing components:

$$G_{j0}(p) = -G_{0j}(p) = \delta^{ab} \frac{4\pi}{s} \epsilon_{0jk} \frac{p^k}{\mathbf{p}^2} \quad (17)$$

The presence of  $p_0$  remains confined in the vertices (13)–(14) and it is trivial because it is concentrated in the Dirac  $\delta$ -functions expressing the momentum conservations. As a consequence, the CS field theory can be considered as a two dimensional model.

First of all we will discuss the one loop calculations. The following observation greatly reduces the number of diagrams to be evaluated:

**Observation:** Let  $G^{(1)}$  be a one particle irreducible (1PI) Feynman diagram containing only one closed loop. Then all the internal lines of  $G^{(1)}$  are either ghost or gluonic lines.

To prove the above observation, we notice that the only way to have a gluonic line preceding or following a ghost line inside a loop is to exploit the ghost-gluon vertex (14). Thus, if a one loop diagram  $G^{(1)}$  with both gluonic and ghost legs exists, the situation illustrated in fig. 1 should occur, in which at least one gluonic tree diagram  $T_{\nu_1 \mu_2 \dots \mu_{n-1} \nu_n}$  is connected to the rest of  $G^{(1)}$  by gluing two of its legs, those carrying the indices  $\nu_1$  and  $\nu_2$  in the figure, to two ghost-gluon vertices  $V_{\text{gh} \nu_1}$  and  $V_{\text{gh} \nu_n}$ . At this point, we recall that these vertices have only spatial components  $V_{\text{gh} i_1}$  and  $V_{\text{gh} i_n}$ ,  $i_1, i_2 = 1, 2$ . As a consequence, since the contractions between gluonic legs are performed with the propagator (17), it is clear that the necessary condition for which the whole diagram  $G^{(1)}$  does not vanish is that  $\nu_1 = \nu_n = 0$ . On the other side, this is not possible, as it is shown by fig. (2). In fact, because of the presence of an  $\epsilon^{\mu\nu\rho}$  tensor in the gluonic vertex (13), the most general gluonic tree diagrams with  $n$  legs  $T_{\nu_1 \mu_2 \dots \mu_{n-1} \nu_n}$  must have at least  $n - 1$  spatial indices in order to be different from zero. This proves the observation. An important consequence is that, at one loop, the only non-vanishing diagrams occur

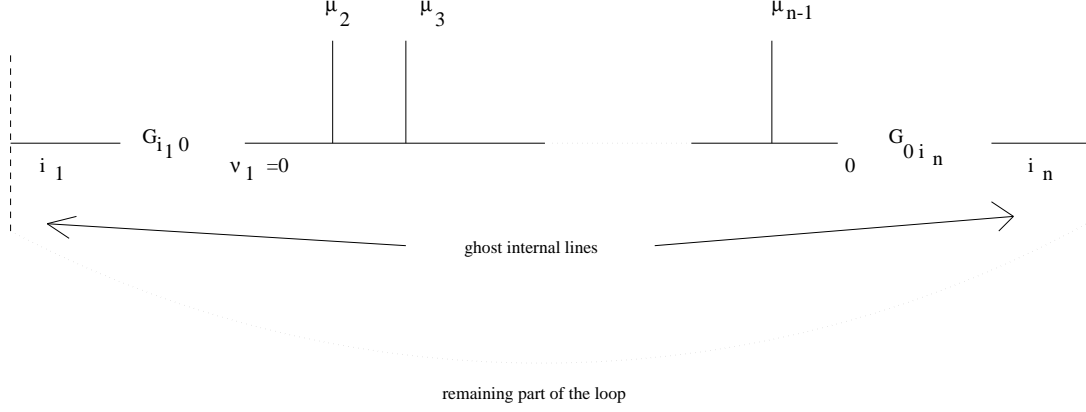


Figure 1: The figure shows the only possible way in which a tree diagram  $T_{\nu_1 \mu_2 \dots \mu_{n-1} \nu_n}$  with  $n$  gluonic legs can be glued to another tree diagram containing also ghost legs in order to build a one loop diagram with mixed ghost and gluonic internal lines.

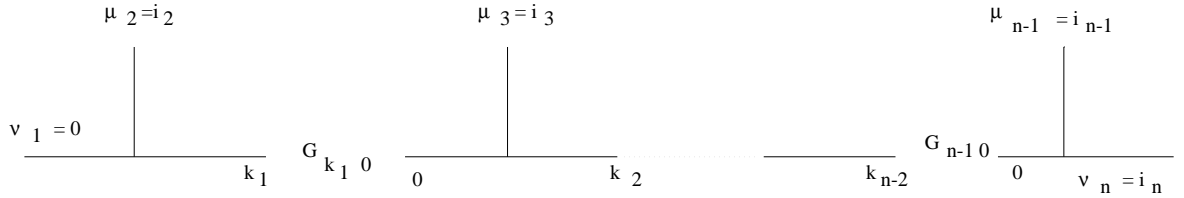


Figure 2: This figure shows that in an arbitrary tree diagram  $T_{\nu_1 \nu_2 \dots \nu_{n-1} \nu_n}$  constructed in terms of the gauge fields propagator (17) and the three gluon vertex (13), only one component in the space-time indices  $\nu_i$ ,  $i = 1, \dots, n$ , can be temporal.





where the constant  $C$  is the result of the integration over the zeroth components of the momenta and it is the same of eq. (18). Apart from an overall sign, eqs. (20) and (21) differ also by the definitions of the momenta. In (20) the variables  $q_2, \dots, q_n$  are in fact given by eq. (19). In eq. (21) we have instead:

$$\begin{aligned} q'_2 &= q'_1 + p_1 \\ &\vdots \\ q'_j &= q'_1 + p_1 + \dots + p_{j-1} \\ &\vdots \\ q'_n &= q'_1 + p_1 + \dots + p_{n-1} \end{aligned} \quad (22)$$

for  $j = 2, \dots, n-1$ .

To compare eq. (21) with (18) and (20) we perform the change of variables

$$q_1 = -q'_1 - p_1 \quad (23)$$

in eq. (21). Exploiting eq. (23) and the relation  $p_1 + \dots + p_n = 0$ , we obtain:

$$\begin{aligned} V_{i_1 \dots i_n}^{a_1 \dots a_n}(2b; p_1, \dots, p_n) &= -C[-i(2\pi)^3]^n \frac{n!(n-1)!}{2} f^{a_1 b'_1 c'_1} f^{a_2 b'_2 b'_1} \dots f^{a_n c'_1 b'_{n-1}} \\ &\delta^{(2)}(\mathbf{p}_1 + \dots + \mathbf{p}_n) \int d^2 \mathbf{q}_1 \frac{q_n^{i_1} q_1^{i_2} \dots q_j^{i_{j+1}} \dots q_{n-1}^{i_n}}{(\mathbf{q}'_1)^2 \dots (\mathbf{q}'_n)^2} \end{aligned} \quad (24)$$

where the variables  $q_2, \dots, q_n$  are now defined as in eq. (19). At this point we can sum eqs. (18), (20) and (24) together. It is easy to realize that the total result is zero, i. e.:

$$V_{i_1 \dots i_n}^{a_1 \dots a_n}(1; p_1, \dots, p_n) + V_{i_1 \dots i_n}^{a_1 \dots a_n}(2a; p_1, \dots, p_n) + V_{i_1 \dots i_n}^{a_1 \dots a_n}(2b; p_1, \dots, p_n) = 0 \quad (25)$$

Still, it is not possible to conclude from eq. (25) that there are no radiative corrections at one loop in C-S field theory. Let us remember in fact that eq. (25) has been obtained from eq. (21) after performing the shift of variables (23). This could be dangerous if there are unregulated divergences. However, it is not difficult to verify that each of the integrals appearing in the right hand sides of eqs. (18), (20) and (21) is IR and UV finite for  $n \geq 3$ . Only the case  $n = 2$  needs some more care. Summing together eqs. (18), (20) and (24) for  $n = 2$ , we obtain the following result:

$$V_{ij}^{ab}(1; p_1, p_2) + V_{ij}^{ab}(2a; p_1, p_2) + V_{ij}^{ab}(2b; p_1, p_2) =$$

$$(2\pi)^6 (2\Lambda_0)^2 N \delta^{ab} \delta^{(2)}(\mathbf{p}_1 + \mathbf{p}_2) \int d^2\mathbf{q} \frac{[q_i(p_1)_j - q_j(p_1)_i]}{\mathbf{q}^2 (\mathbf{q} + \mathbf{p}_1)^2} \quad (26)$$

where we have put  $q'_1 = q_1 = q$ . As we see, the integrand appearing in the rhs of (26) is both IR and UV finite. Moreover, a simple computation shows that the integral over  $\mathbf{q}$  is zero without the need of the shift (23). As a consequence, there are no contributions to the Green functions at one loop.

Now we are ready to consider the higher order corrections. At two loop, a general Feynman diagram  $G^{(2)}$  can be obtained contracting two legs of a tree diagram  $G^{(0)}$  with two legs of a one loop diagram  $G^{(1)}$ . As previously seen, the latter have only gluonic legs and their tensorial indices are all spatial. Consequently, in order to perform the contractions by means of the propagator (17), there should exist one component of  $G^{(0)}$  with at least two temporal indices, but this is impossible. To convince oneself of this fact, it is sufficient to look at fig. (2) and related comments. The situation does not improve if we build  $G^{(0)}$  exploiting also the ghost-gluon vertex (14), because it has no temporal component. As a consequence, all the Feynman graphs vanish identically at two loop order. Let us notice that it is possible to verify their vanishing directly, since the number of two loop diagrams is relatively small in the Coulomb gauge and one has just to contract the space-time indices without performing the integrations over the internal momenta. However, this procedure is rather long and will not be reported here.

Coming to the higher order computations, we notice that a diagram with  $N + 1$  loops  $G^{(N+1)}$  has at least one subdiagram  $G^{(N)}$  containing  $N$ -loops. Supposing that  $G^{(N)}$  is identically equal to zero because it cannot be constructed with the Feynman rules (12)–(14) and (17), also  $G^{(N+1)}$  must be zero. As we have seen above, there are no Feynman diagrams for  $N = 2$ . This is enough to prove by induction that the C–S field theories have no radiative corrections in the Coulomb gauge for any value of  $N$ .

## 4 Conclusions

In this paper we have proved with explicit computations that the C–S field theories do not have quantum corrections in the Coulomb gauge. At two loop order and beyond, this is a trivial consequence of the fact that it is impossible to construct nonzero Feynman diagrams starting from the vertices and propagators given in eqs. (12)–(14) and (17). At one loop, instead, nontrivial

cancellations occur between the different diagrams. We have also seen that the perturbative expansion of the Green functions is not affected by UV or IR divergences. Only the spurious singularities are present, which are related to the fact that the propagators are undamped in the time direction. They are similar to the singularities observed in the four dimensional Yang–Mills field theories [12], but in the C–S case appear in a milder form. In fact, after the regularization (16), their contribution at any loop order reduces to a factor in the radiative corrections and does not influence the remaining calculations. Therefore, the results obtained here are regularization independent. Moreover, the vanishing of the quantum contributions described in Section 3 is a peculiarity of the Coulomb gauge that does not strictly depend from the fact that the C–S field theories are topological. An analogous situation occurs in the light cone gauge in the presence of a boundary. In that case, radiative corrections arise due to the interactions of the fields with the boundary, but each Feynman diagram corresponding to these interactions vanishes identically [23].

In summary, our study indicates that the Coulomb gauge is a convenient and reliable gauge fixing, especially in the perturbative applications of C–S field theory. Let us remember that, despite of the fact that the theory does not contain degrees of freedom, the perturbative calculations play a relevant role, for instance in the computations of knot invariants [21], [24]–[27]. Contrary to what happens using the covariant gauges, where it becomes more and more difficult to evaluate the radiative corrections as the loop number increases [21, 25, 28], in the Coulomb gauge only the tree level contributions to the Green functions survive. This feature is particularly useful in the case of non-flat manifolds, where the momentum representation does not exist. For instance, Feynman rules analogous to those given in eqs. (8)–(14) have been derived also on the compact Riemann surfaces [29]. In the future, besides the applications in knot theory, we plan to extend our work also to C–S field theories with non-compact gauge group, in order to include also the theory of quantum gravity in  $2 + 1$  dimensions. Moreover, most of the pathologies that seem to afflict the four dimensional gauge field theories, like spurious and infrared divergences, are also present in the C–S field theories, but in a milder form. As a consequence, the latter can be considered as a good laboratory in order to study their possible remedies. For example, it would be interesting to apply to the Yang–Mills case the regularization (16) introduced here for the spurious singularities. Let us notice that a differ-

ent regularization has been recently proposed in [15]. Finally, the present analysis is limited to the pure C–S field theories and more investigations are necessary for the interacting case. Until now, only the models based on abelian C–S field theory have been studied in details, in particular the so-called Maxwell-Chern-Simons field theory, whose consistency in the Coulomb gauge has been checked with several tests [9].

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